PH16212, Homework 6

Deadline Dec. 2, 2019

1. (Generalized Unitarity). Consider the one-loop color-ordered gluon amplitude $A^{(1)}(1^-2^+3^+4^-)$. The inverse propagators are

$$D_1 = l^2, \quad D_2 = (l-p_1)^2, \quad D_3 = (l-p_1-p_2), \quad D_4 = (l+p_4)^2 \tag{1}$$

From our class, we know that there are two 4D maximal cut solutions. The first solution has the spinor decomposition,

l^*	$x1$	$\tilde{4}$
$l^* - p_1$	1	$x\tilde{4} - \tilde{1}$
$l^* - p_1 - p_2$	$y1 - 2$	$\tilde{2}$
$l^* + p_4$	$x1 + 4$	$\tilde{4}$

(2)

with $x = -\frac{\langle 43\rangle}{\langle 13\rangle}$, $y = -\frac{[14]}{[24]}$. The second solution is

l^*	$x4$	$\tilde{1}$
$l^* - p_1$	$x4 - 1$	$\tilde{1}$
$l^* - p_1 - p_2$	2	$y\tilde{1} - \tilde{2}$
$l^* + p_4$	4	$x\tilde{1} + \tilde{4}$

(3)

with $x = -\frac{s\langle 13\rangle}{(s+t)\langle 34\rangle}$, $y = -\frac{t\langle 13\rangle}{(s+t)\langle 23\rangle}$. On these solutions, the amplitude $A^{(1)}(1^-2^+3^+4^-)$ factorizes as the product of four tree amplitudes.

- On each solution, calculate the tree amplitudes product and then simplify it.
- Find the box coefficient of $A^{(1)}(1^-2^+3^+4^-)$.

2. Define $I = \langle y^2 - x^3 - 1, x^2 + 2y^2 - 1\rangle$ and $S = \mathcal{Z}(I)$.

- In the "DegreeReverseLexicographic" ordering with $x \succ y \succ z$, calculate the Groebner basis of I.
- Define $F = x^3 + y^3$. In the "DegreeReverseLexicographic" ordering with $x \succ y \succ z$, compute the companion matrix of F. Analytically compute

$$\sum_{p\in S} F(p)\,. \tag{4}$$

Check it with the numeric computation.

3. Define $I = \langle x^2 + y^2 + z^2 - 1, xyz - 3, z^3 - x^2 - 2y \rangle$. Eliminate y and z, to find the generating polynomial $f(x) \in \mathbb{Q}[x] \cap I$.